Plato's Fractal Line: A Solution to the Overdetermination Problem

This paper argues, first, that Plato's Divided Line, found at the conclusion of Republic VI, is best explicated by means of modern fractal geometry, and second, that such a fractal-based understanding of the line resolves the so-called overdetermination problem. This exegetical paradox arises from the fact that the middle two subsections of the line appear simultaneously both equal and unequal. On the one hand, the middle subsections are mathematically equal.[1] On the other hand, however, each subsection represents ascending, and thus unequal, degrees of ontological reality and epistemic clarity (509d; 511d). Building upon recent work, such as that of Nicholas Smith (Smith 1981), I propose a new solution to this contradiction.

I begin with the essentials of Socrates' description of the Divided Line. Socrates asks Glaucon to imagine a line "divided into two unequal sections" (509d6) and then to "divide each section … in the same ratio as the line" (509d6-8):

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     1            2              2                   4
[----------|--------------------|--------------------|----------------------------------------]
A   (x)    B      (y)        C       (a)        D                 (b)                    E
```

As Socrates explains, the lower half of the line (AC) represents the physical realm, while the upper half (CE) pictures the intellectual realm. Within the physical realm, the smallest subsection (AB) is comprised of shadows and images (509e1), while its counterpart (BC) contains the re-

[1] Foley 2008, 2 provides an arithmetic proof:

x/y = a/b = (x+y)/(a+b)	[solve for x]
x = ay/b	[substitute for x]
a/b = ((ay/b)+y)/(a+b)	[multiply by (a+b)]
a(a+b)/b = (ay/b)+y	[cross-multiply the right side]
a(a+b)/b = (ay+by)/b	[multiply out b and extract the common y]
a(a+b) = y(a+b)	[divide out (a+b)]
a = y	

spective objects of these shadows and images (510a3). Within the intellectual realm, subsection CD consists in intellectual abstractions of physical objects (510d2; 510b3-4) and the uppermost subsection (DE) represents the Forms (510b9). Upon this complex mathematical and metaphorical structure, Plato's Socrates encodes an ontology and an epistemology (511d-e). The overdetermination problem thus has far-reaching consequences for our understanding of key elements of Plato's philosophy.

In order to confront this problematic conundrum, I first argue that the Divided Line is a semi-fractal. In simplest terms, a fractal is a self-similar mathematical pattern, something that "contains copies of itself at many different scales" (Falconer 2007, xviii; see also Mandelbrot 1977 and Mandelbrot 1983). The strict definition of a fractal contains at least 5 necessary conditions:

[1] a fractal is self-similar;

[2] has a fine structure (i.e. retains its detail at any scale),

[3] has a simple definition,

[4] is obtained by a recursive procedure, and

[5] is awkward to describe its local geometry.[2]

I argue that Socrates' initial description of the mathematical structure of the line (509d6-8) meets 4 of these criteria; however, Socrates' divided line does not present a fine structure (criterion 2). Rather, its details are lost after the second iteration of the function. And yet, the mathematical apparatus does offer the possibility of a full fractal. One need only continue Socrates' described recursive procedure ad infinitum to produce a full fractal:

[2] Falconer 2007, xviii adds, [6] "the geometry ... is not easily described in classical terms" and [7] "its size is not quantified by the usual measures such as length," although I find these to be sufficiently covered by [5].

“Divided Line Function” repeated

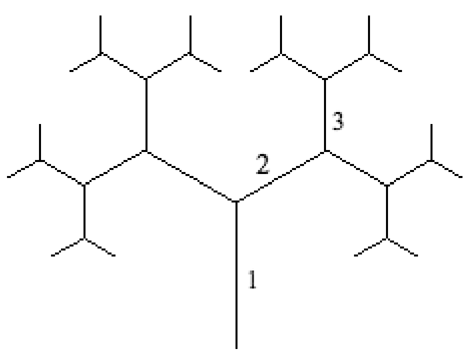
```
                                             81
---------------------------------------------------------------------------------------------   D^0
      9              18                  18                         36
-----------|--------------------|--------------------|-------------------------------------   D^1

1 2  2   4 | 2   4    4     8   | 2   4    4     8    | 4      8       8           16
-|--|--|----I--|----|----|--------I--|----|----|--------I----|--------|--------|----------------   D^2
```

[*ad infinitum*]

In short, Plato’s Divided Line represents a semi-fractal. Socrates establishes the mathematical apparatus for creating a fractal, but only uses this apparatus to construct an image with two degrees of complexity. Nonetheless, using the concept of a fractal as a heuristic device can, I argue, indicate an answer to the overdetermination problem.

Insofar as Plato creates a quasi-fractal, an interpreter ought to see the central sections as both equal and unequal. There is, however, a distinction between an absolute equality and a contextual inequality. Both subsections contain the same stuff, but the mind views that material in different aspects. Fractals elucidate how such an absolute equality and an aspectual inequality can coexist. Consider a simple tree fractal:



Line segment 2 is both a “l usly a “trunk” for line segment 3. I argue that the middle of Plato’s Divided Line functions just as line segment 2:

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                          The Fractal Line

                    D          The Forms              E
            -------------------|---------------------------------------]
 Shadows     Objects
[----------|-------------------|
A          B                   C
```

While the material is the same (an absolute equality), one can see that material in two ways (an aspectual inequality): [a] as objects of knowledge or [b] as means to knowledge. Since Plato holds that physical objects are not the true objects of knowledge, the latter proves more valuable.

In sum, this paper demonstrates that the overdetermination problem is solvable and actually points to one of the key ideas encoded in the mathematics of the Divided Line. The middle two subsections, made up of physical objects and their intellectual abstractions, are indeed simultaneously equal and unequal, and that’s the point. For knowledge to be attainable yet also appropriately difficult, physical objects must represent both the imaged objects of shadows and the imaging objects of Forms.

Works Cited

Falconer, Kenneth. 2007. Fractal geometry: mathematical foundations and applications. Wiley.

Foley, Richard. 2008. “Plato’s Undividable Line: Contradiction and Method in Republic VI.” JHP 46.1: 1-23.

Mandelbrot, Benoit B. 1983. The Fractal Geometry of Nature. Macmillan.

- - -. 1977. Fractals: form, chance, and dimension. WH Freeman.

Smith, Nicholas. 1981. “The Objects of ‘Dianoia’ in Plato’s Divided Line.” Apeiron 15.2: 129-37.