Inventing Incommensurability: Traces of a Scientific Revolution in Early Greek Mathematics in the Times of Plato

Most of early Greek mathematics lies in darkness: though it might have begun with Thales as early as at the beginning of the 6th century BCE, the first *authentic* testimonia date from the first half of the 4th century BCE. All earlier testimonia come from later authors who were (re-)writing them in the terms of (post-)Euclidean mathematics.

Though not a mathematician in the strict sense himself, one of the first authors to give an undistorted glimpse of how early Greek mathematics looked like is Plato: his works contain the earliest reliable direct and authentic references to Greek mathematics, and this in great number.

Among them, there is an odd passage in the *Laws* regarding incommensurability, that is two specific mathematical magnitudes' not having a common measure with each other (819d–820b): for since the 'Athenian' tells 'Kleinias' that he only very recently had heard of this phenomenon for the very first time, it is implied that its discovery took place not until some time in the first half of the 4th century BCE (cf. Ps.-Plato, *Sisyphus* 388e). However, incommensurability is, following numerous ancient testimonia, traditionally believed to have been discovered a long time before that, that is, by the 5th or even 6th century BCE Pythagoreans. Plato, therefore, evidently must be either wrong or not – but despite a raging and controversial scholarly debate, no consensus could have been reached yet (cf. Cuomo 2001, 17; 30; and, in particular see Szabó 1978; Knorr 1975; Zhmud 1997; Fowler 1999).

This paper will try to reconcile the scholarly factions by taking a fresh look at the evidence from the viewpoint of the contemporary history and philosophy of science, in particular Thomas Kuhn's theory of scientific revolutions (Kuhn 1986). It shall be proposed that incommensurability proper in the strict, mathematical sense was indeed discovered only as late as in the 4^{th} century BCE, presumably by the mathematician Theaetetus – which, by the way, would square well also with another quite odd testimonium for early Greek mathematics in Plato's works, that is, the famous discussion of incommensurable numbers in the *Theaetetus* (cf. Burnyeat 1978).

If this be right, incommensurability had not been known before Theaetetus nor had there been, *a fortiori*, any theoretical explanation of it. Rather, the only thing mathematicians might have recognized before that point in time would be that with regard to some specific magnitudes there is some more or less severe difficulty (though, principally, no impossibility) to find a common measure, this endeavor being part of the research program of the then 'normal science.'

Such a state of mathematical theory, however, would be quite different from and, in a certain sense, contrary to recognizing and proving that some magnitudes are *principally* never commensurable with some other magnitudes. For such an insight was, then, only possible in the framework of Eudoxus's revolutionary theory of proportion: only this approach allowed for formulating a full-fledged theory of incommensurability and thus, at the same time, for effectively *inventing* (rather than 'discovering') incommensurability as a mathematical phenomenon *sui generis*.

Nonetheless – that is, in order to explain the ancient testimonia regarding the alleged contribution made by the early Pythagoreans –, in hindsight and presupposing a linear progress of science as it was common in antiquity (cf., e.g., Aristotle's history of philosophy), it might indeed have appeared that the concept of incommensurability had already been known before Eudoxus and Theaetetus – just because post-Eudoxean mathematicians dealt with the (seemingly) same data as the pre-Eudoxean mathematicians. However, the theoretical frameworks before and after Eudoxus would have been incommensurably different, for

Eudoxus's theory of proportion brought about a fundamental paradigm shift which led to the possibility of a revolutionary re-interpretation of the already well-known data.

By reassessing the history of the discovery of the concept of mathematical incommensurability from a contemporary history and philosophy of science viewpoint, this paper will help to illuminate the history of a central mathematical phenomenon in ancient Greek mathematics and, thus, contribute to a better understanding of the overall history of early Greek mathematics, including Plato's relationship to and role in it. This will at the same time shed light on the riddle-laden general question of what was so special about classical Greek mathematics.

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