

Greeks Doing Algebra

There is a long-standing consensus in the history of mathematics that geometry came from ancient Greece, algebra came from medieval Persia, and the two sciences did not meet until seventeenth-century France (e.g. Bell 1945). Scholars agree that the Greek mathematicians had no methods comparable to algebra before Diophantus (3rd c. CE) or, many hold, even after him (e.g. Szabó 1969, Unguru and Rowe 1981, Grattan-Guinness 1996, Vitrac 2005. For a survey of arguments see Blåsjö 2016). The problems that we would solve with algebra, the Greeks, especially the authors of the canonical and most often studied works (such as Euclid and Apollonius of Perga), approached with spatial geometry or not at all. This paper argues, however, that the methods which uniquely characterize algebra, such as information compression, quantitative abstraction, and the use of unknowns, do in fact feature in Greek mathematical works prior to Diophantus. We simply have to look beyond the looming figures of Hellenistic geometry. In this paper, we shall examine three instructive cases of algebraic problem-solving methods in Greek mathematical works before Diophantus: The Sand-reckoner of Archimedes, the *Metrica* of Hero of Alexandria, and the *Almagest* of Ptolemy.

In the *Sand-reckoner*, Archimedes develops a system for expressing extremely large numbers, in which the base unit is a myriad myriad. His process is indefinitely repeatable, and theoretically scalable to express a number of any size. Simple though it sounds to us, this bit of information compression, by which a cumbersome quantity is set to one in order to simplify notation and computation, is a common feature of modern mathematics but was almost alien to the Greeks. In the *Metrica*, Hero solves a series of geometrical problems using quantitative abstraction, in which continuous magnitudes such as straight lines are treated as though composed of countable units. With this step, Hero is also able to obtain numerical values for

unknown quantities (such as the area of a triangle) through basic arithmetical operations.

Ptolemy, calculating the table of chords in Book I of the *Almagest*, performs all three of these algebraic problem-solving methods. In his introduction to the table of chords, he states that he is employing these methods in order to simplify and systematize the calculation process.

The three examples of algebraic methods outlined in this paper show that the essentials of algebra, both in itself and in its application to geometry, were in use long before Descartes, al-Khwarizmi, and even Diophantus. In addition, they show that the canonical texts of Hellenistic geometry were not the only places where complex and sophisticated mathematics was being done.

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